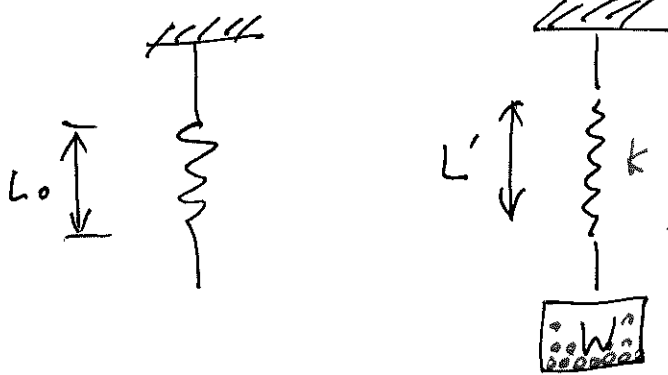


Chapter 3 Mechanical properties of Materials

- Constitutive law (solid mechanics)
 \Rightarrow relationship b/w stress & strain

• physics

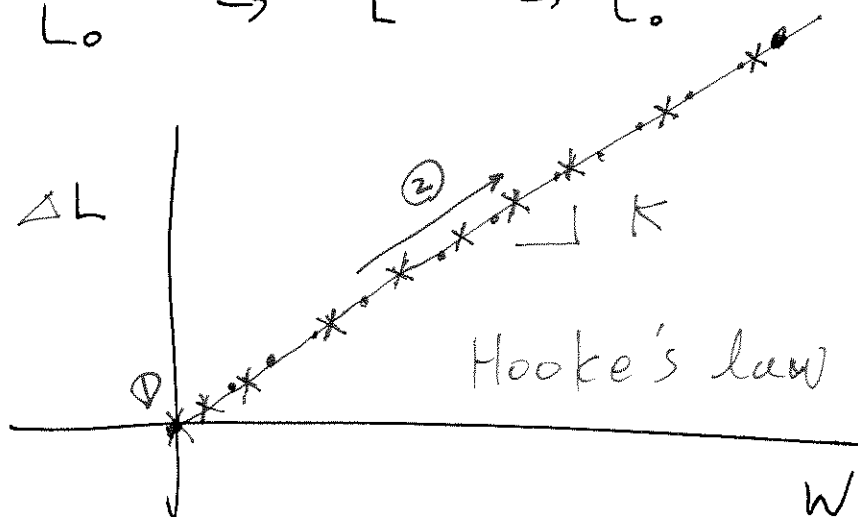
Force vs. Deformation



Hooke's law
 $W = k(L' - L_0)$
 k : spring constant

- ① initial L_0 ② load L' ③ unload L_0

Linear Elastic (assumption)



$$(\Delta L = L' - L_0)$$

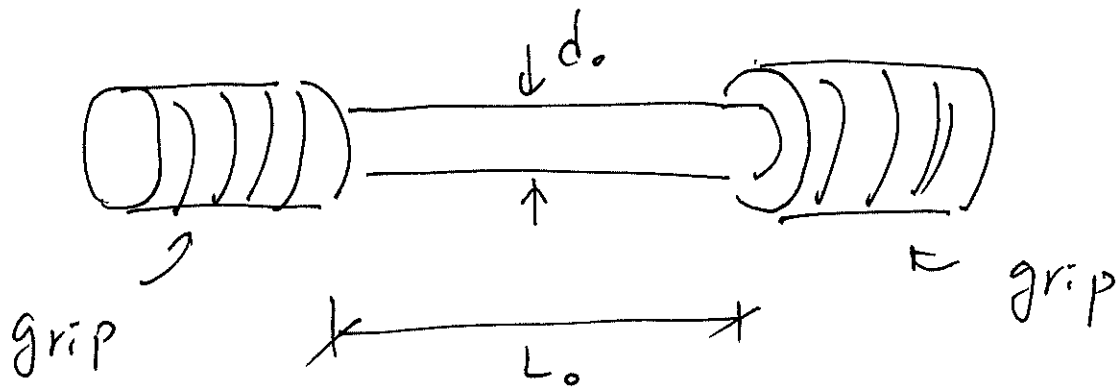
- \bullet : loading
- \times : unloading

3.1 Tension & Compression Test

(Uniaxial)

Refer to ASTM E8 ✓

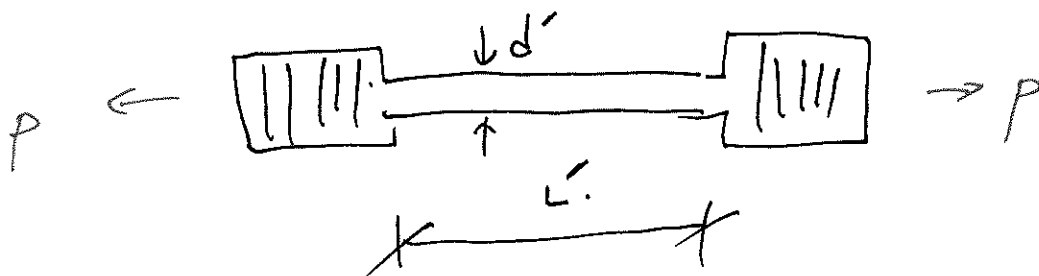
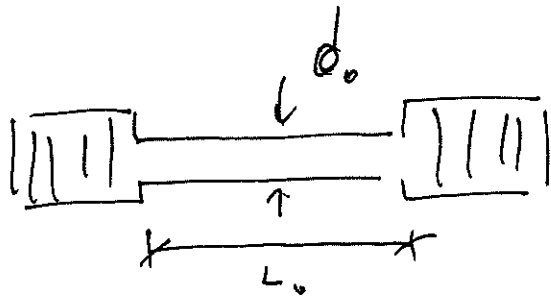
Uniaxial Tension Test (e.g. steel)



Typical size

$$d_0 \sim \frac{1}{2} \text{ in}$$

$$L_0 \sim 2 \text{ in}$$

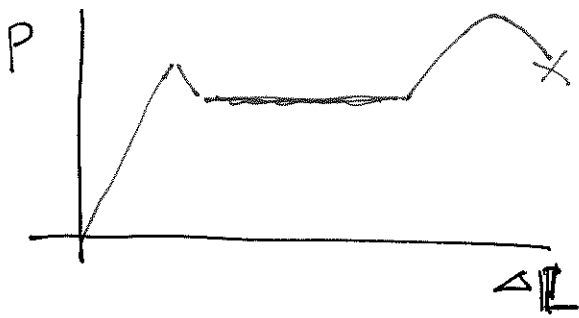


Load: P

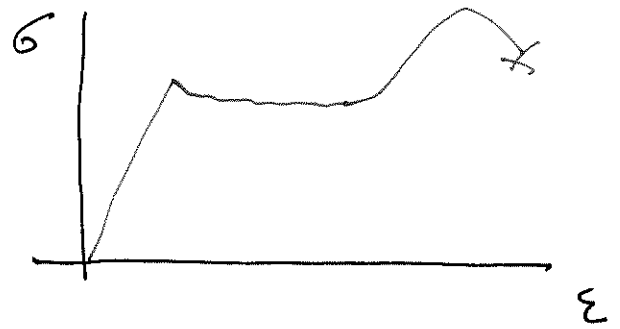
Deform: $\Delta L = L' - L_0$

$$\sigma_{avg} = \frac{P}{\frac{\pi}{4} d_0^2}$$

3.2 Stress - Strain Diagram (Uniaxial Tension Test)



⇒



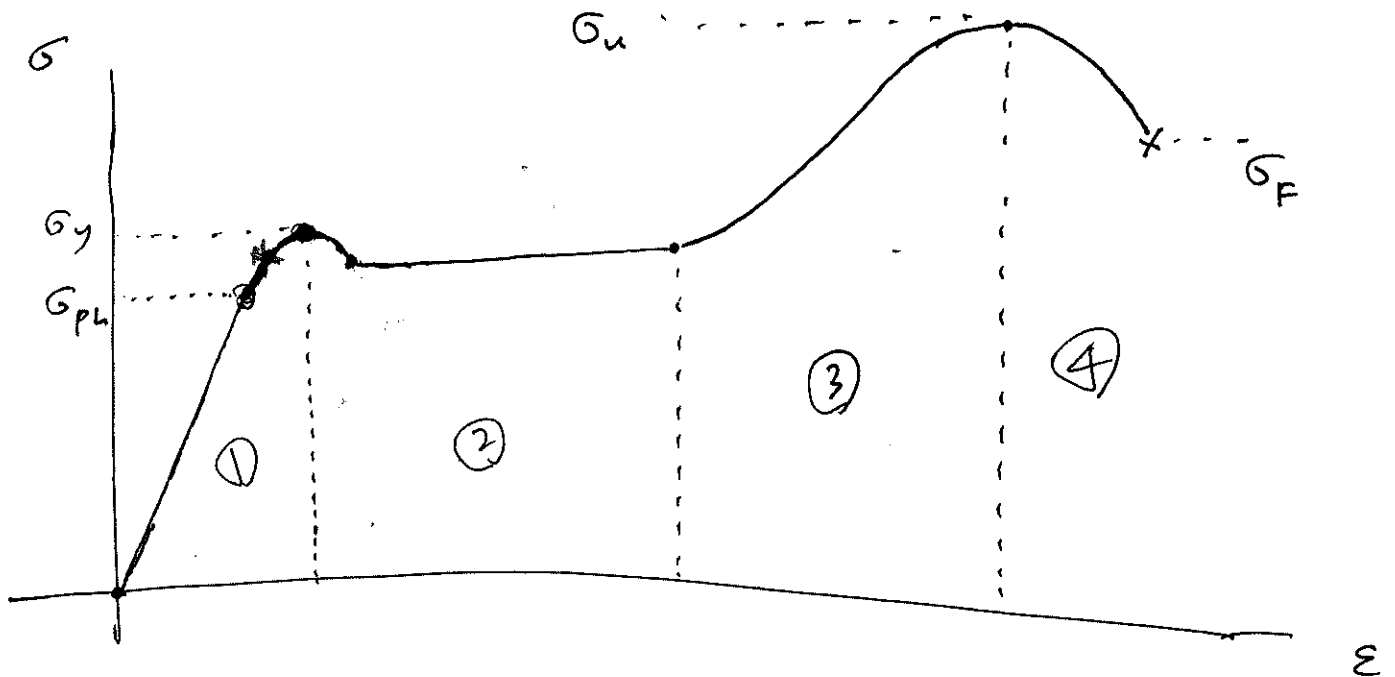
$P - \Delta L$
(force) vs. (Deform.)

⇒

$\sigma - \epsilon$

$$\left(\begin{array}{l} \sigma_{avg} = \frac{P}{A_0} \\ \epsilon_{avg} = \frac{\Delta L}{L_0} \end{array} \right) \quad \left(A_0 = \frac{\pi}{4} d_0^2 \right)$$

$\sigma - \epsilon$ diagram for a uniaxial tension test (Steel)



① Elastic Behavior

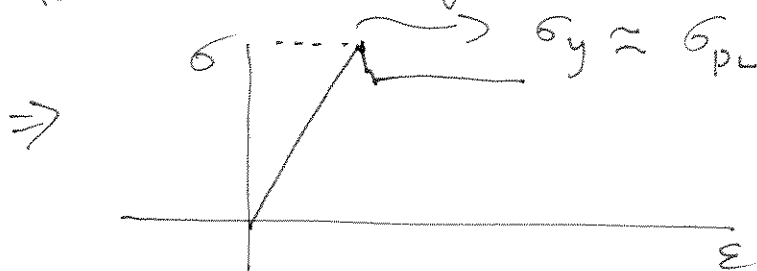
σ_{PL} : proportional limit. ^{material}
 \Rightarrow Below σ_{PL} : ~~is~~ is linear elastic

σ_y : yield point

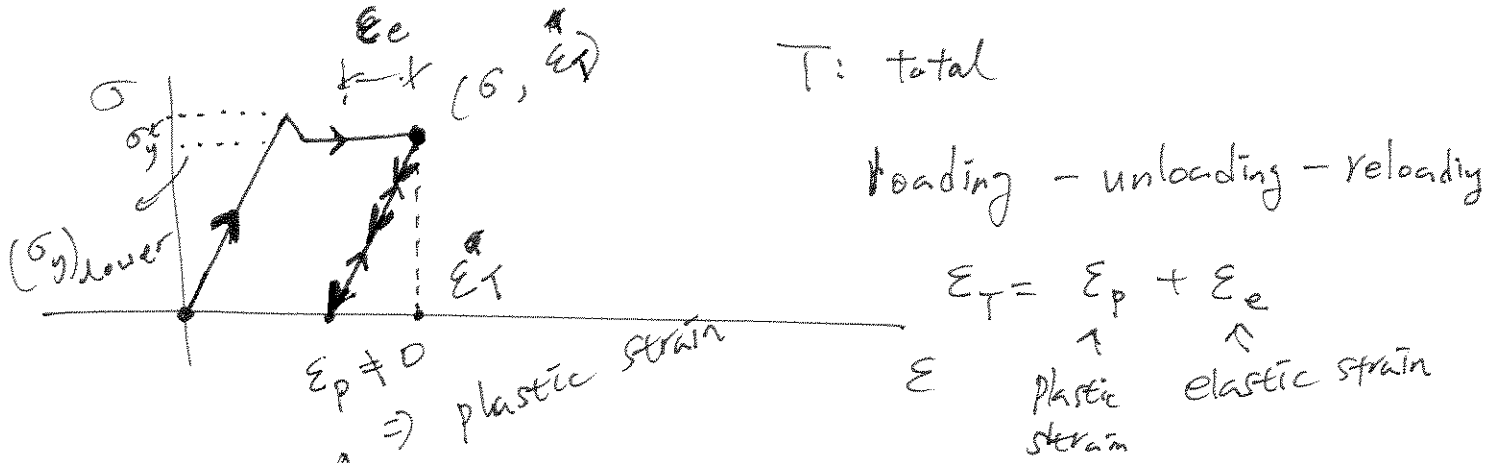
up to σ_y : material is elastic

b/w σ_{PL} and σ_y : material is non-linear elastic

In some material, σ_y and σ_{PL} are close enough to be interchangeable



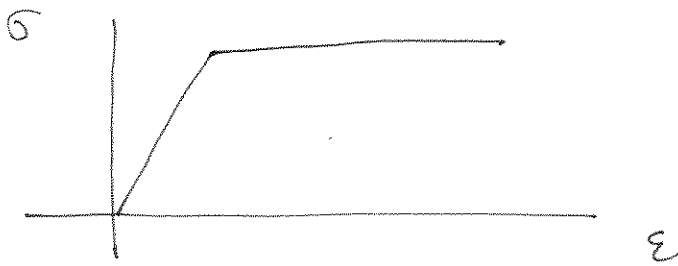
② yielding (plastic deformation)



specimen is now "permanently" deformed
 \Rightarrow plastic deformation occurred

■ σ_y : a peak followed by a lower yield point which usually close to σ_y

$\sigma_y \approx (\sigma_y)_{lower}$
 $\Rightarrow \sigma_y \gg (\sigma_y - (\sigma_y)_{lower})$



~~(elasto-perfect plastic)~~

■ In segment ②, material deforms plastically.
 \Rightarrow permanent deformation

02/21/2022

③ Strain hardening

• After yield (②), material can support additional load (stress increases)

• Increases to σ_u , ultimate stress

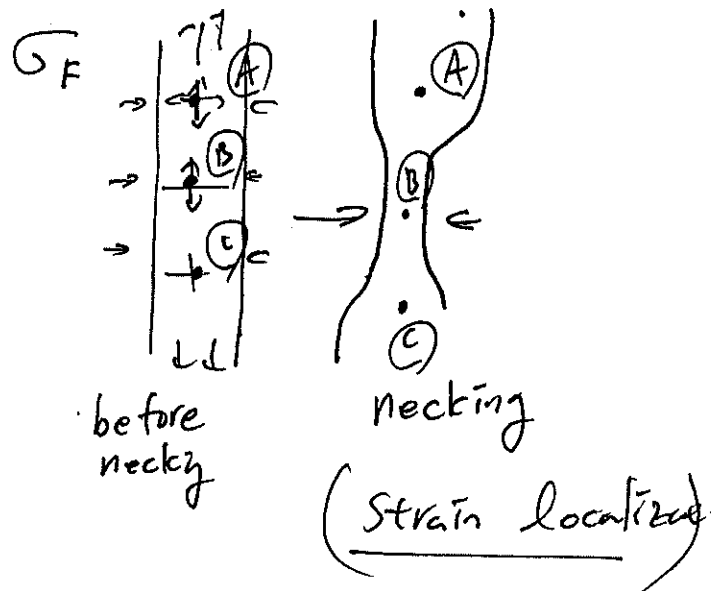
• σ_u : max. stress that the material can sustain

④ Necking (strain softening)

• ~~Beyond~~ ^{Before} σ_u , specimen is deforming uniformly

• Beyond (or Past) σ_u , cross-section ~~is~~ decreases in a specific region (\Rightarrow necking)

• Specimen breaks at



Constitutive Law

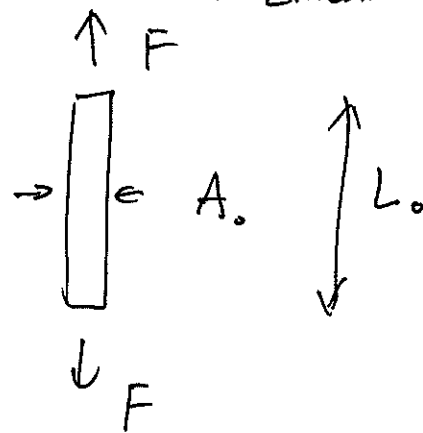
~~σ~~ vs. ~~ϵ~~ relationship
Stress Strain

ex) Hooke's law ($F = k \Delta L$)

↳ Linear Elastic Spring

$$F = k \Delta L$$

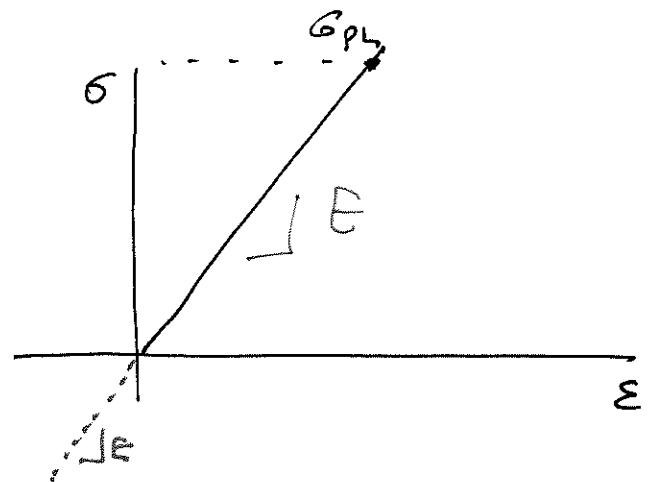
$$\left(\frac{F}{A_0} = \sigma \right) \quad \left(\epsilon = \frac{\Delta L}{L_0} \right)$$



$$\sigma A_0 = k (\epsilon L_0)$$

$$\sigma = \left(\frac{k L_0}{A_0} \right) \epsilon$$

$$\boxed{\sigma = E \epsilon}$$



Where E is Young's modulus of material